Unit 4
Day 4
Section 8.4
The Binomial Theorem

BINOMIAL COEFFICIENT

For non-negative integers n and r, with $r \leq n$,
the symbol $\binom{n}{r}$ is defined as

$$\binom{n}{r} = \frac{n!}{r!(n-r)!}$$

Using the TI-30X to calculate the binomial coefficient we use the $\boxed{\text{nCr}}$ feature.

$$\binom{10}{7} = \frac{10!}{7!3!} = \frac{10 \cdot \overset{3}{\cancel{9}} \cdot \overset{4}{\cancel{8}} \cdot \cancel{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}}{\cancel{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} \cdot \underset{1}{\cancel{3}} \cdot \underset{1}{\cancel{2}} \cdot 1} = 120$$

USE YOUR CALCULATOR TO DO THE FOLLOWING:

10 , PRB, ->, nCr, =, 7, =   This should give you 120.

Evaluate.

1) $\frac{7!}{3!4!} = \frac{7 \cdot \cancel{6} \cdot 5 \cdot \cancel{4 \cdot 3 \cdot 2 \cdot 1}}{\cancel{3} \cdot \cancel{2} \cdot 1 \cdot \cancel{4 \cdot 3 \cdot 2 \cdot 1}} = 35$

2) $\binom{8}{3} = \frac{8!}{3!\,5!} = \frac{8 \cdot 7 \cdot \cancel{6} \cdot \cancel{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}}{\cancel{3} \cdot \cancel{2} \cdot 1 \cdot \cancel{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}} = 56$

$\binom{8}{3} = {}_8C_3$

3) ${}_{100}C_2 = 4950$

$\binom{100}{2} = \frac{100!}{2!\,98!} = \frac{\cancel{100}^{50} \cdot 99}{\cancel{2} \cdot 1} = 4950$

BINOMIAL THEOREM

For any positive integer n and any complex numbers x and y ,

$$(x+y)^n = \sum_{r=0}^{n} \binom{n}{r} x^{n-r} y^r$$

Refer to p. 603 for Binomial Theorem

USING THE BINOMIAL THEOREM

Write the binomial expansion for each expression.

4)

$(p+q)^5$

$\binom{5}{0}p^5q^0 + \binom{5}{1}p^4q^1 + \binom{5}{2}p^3q^2 + \binom{5}{3}p^2q^3 +$

$+ \binom{5}{4}pq^4 + \binom{5}{5}p^0q^5$

$p^5 + 5p^4q + 10p^3q^2 + 10p^2q^3 + 5pq^4 + q^5$

5)

$(x-2y)^6$

$$\binom{6}{0}x^6(-2y)^0 + \binom{6}{1}x^5(-2y) + \binom{6}{2}x^4(-2y)^2 + \binom{6}{3}x^3(-2y)^3$$

$$+\binom{6}{4}x^2(-2y)^4 + \binom{6}{5}x(-2y)^5 + \binom{6}{6}x^0(-2y)^6$$

$$(1)x^6 + (6)x^5(-2y)$$

$$x^6 - 12x^5y + 60x^4y^2 - 160x^3y^3 + 240x^2y^4 - 192xy^5 + 64y^6$$

HOMEWORK

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